The Radiative Heat Exchange Model Component

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This document describes the Radiative Heat model component (RHEComponent) computes the shortwave and longwave radiation contributions to stream temperature. The RHEComponent was developed to be primarily used within the HydroCouple component-based modeling framework (Buahin and Horsburgh, 2016). However, it can be compiled and executed as a standalone executable.

# Formulations

The 1D advection dispersion heat transport equation that is solved by the RHEComponent model is shown in Equation 1.

(1)

where is the water temperature , is the time , is the velocity of the water in the channel , is the distance along the channel , longitudinal dispersion , is the water density , is the specific heat capacity of water , is the temperature of the water , are external radiant heat fluxes incident on the water surface, are heat supplied by other external sources , is the depth of water in the channel . Equation 1 is approximated numerically using the finite volume method as shown subsequently. The integral version of Equation 1 over a time step from to over the control volume i (i.e., CVi in Figure 1) is shown in Equation 2.



Figure 1. 1D control volume

(2)

where is the volume of the CV , represents the current time step , and represents the next time step where we seek a solution. Using Gauss’s divergence theorem and expanding the terms for Equation 3 yields:

(3)

where represents the number of inlet and outlet boundaries for the CV, represents summation of the advective heat fluxes across the inlet and outlet boundaries of the CV, represents the sum of the dispersive heat fluxes across the inlet and outlet boundaries of the CV, and is the cross sectional of flow. Using an explicit time marching approximation for the CV depicted in Figure 1 yields Equation 4, which is expands to Equation 5.

(4)

where fluxes out of the CV take on positive values, fluxes into the CV take on negative values, values with the superscripts and represent values at the current time step and next time step respectively, values with the subscripts , , and represent values at the current CV, its left boundary, and right boundary respectively, is the time step , and is the flow for the CV .

External sources of heat fluxes, including latent heat from evaporation and condensation as well as sensible heat exchanges from conduction and convection with the atmosphere can be specified in the input file or retrieved from other models that are coupled to the RHEComponent.

## Advection

Several methods are available for discretizing the advection terms in Equation 5. These include the upwind, central and hybrid differencing methods. Additionally, several total variation diminishing (TVD; Harten, 1983) schemes are also available for problems that have sharp discontinuities in their solution domain. An exhaustive treatment of TVD schemes is provided by Versteeg and Malalasekera (2007) and are not described here.

For the first-order accurate upwind differencing scheme, the assumptions made for inlet and outlet advective heat fluxes for boundaries of the control volume are prescribed as follows:

(5)

(6)

For the second-order accurate central differencing scheme, the inlet and outlet advective heat fluxes at the boundaries of the control volume are interpolated using the inverse distance weighting (IDW) interpolation scheme as shown in equations 7 and 8.

(7)

(8)

While the upwind differencing scheme is stable, it is only first order accurate, which gives rise to false diffusion. This contrasts with the central differencing scheme, which although second-order accurate, does not possess the transportiveness property (i.e., ability to account for flow direction as well as the upwind scheme especially for highly advective flows) (Versteeg and Malalasekera, 2007). The hybrid differencing scheme proposed by Spalding (1972) attempts to split these tradeoffs be assessing whether advection or dispersion is the dominant transport mechanism. The hybrid differencing scheme proceeds by first estimating the Peclet number at the face of the control volume of interest as follows:

(9)

The flux through that face of the control volume is then estimated as follows:

for (10)

for (11)

for (12)

where and are the IDW interpolation factors for the current and left control volumes that surround the boundary under consideration respectively.

## Dispersion

The spatial gradients of temperature at inlet and outlet of the CV used for computing dispersion in Equation 4 is discretized numerically as follows:

(13)

(14)

Following the QUAL2K model, the RHEComponent adopts the formulations by Fischer *et al.* (1979) to calculate longitudinal dispersion when it is not explicitly provided as follows:

(15)

where is the channel width , is the mean flow depth , and is shear velocity of the CV. The shear velocity is calculated as:

(16)

where is the channel slope. The computed dispersion coefficient is compared with the numerical dispersion estimated using Equation 17.

(17)

If the computed numerical dispersion is less than the computed dispersion in Equation 17, is used as the dispersion coefficient used in Equation 4 otherwise, the dispersion coefficient is set to zero.

## Evaporation and Condensation Heat Fluxes

Evaporation/condensation is a function of the sensible heat carried with the evaporated water, the latent heat of evaporation, density of water, and the evaporative rate as expressed in Equation 18 (Webb and Zhang, 1997; Evans *et al.*, 1998; Boyd and Kasper, 2003).

(18)

where is the latent heat of vaporization and E is the evaporative rate . The latent heat of vaporization is estimated as a weak function of water temperature using Equation 19 (Martin and McCutcheon, 1998).

(19)

where is the water temperature in the channel ().

Several approaches are available to estimating the evaporative rate including mass transfer methods, explicit energy balance methods, and combination methods that combine both mass and energy balance methods. In the RHEComponent, a mass balance method is implement. However, work is ongoing to incorporate other methods. Following Dingman (2008), the evaporative rate is estimated using Equation 20.

(20)

where is the saturation vapor pressure of the evaporating surface , is the actual vapor pressure , and is a wind function used to estimate the adiabatic portion of evaporation (Boyd and Kasper, 2003). is computed using Equation 21 (Raudkivi, 1979; Chapra, 2008).

(21)

The actual vapor pressure () is calculated as a function of relative humidity and saturation vapor pressure ( using Equation 22.

(22)

where is computed using equation 23.

(23)

where is air temperature in .

Extensive observations have yielded Equation 24 as the general form of the wind function (Martin and McCutcheon, 1998; Shanahan *et al.*, 1984).

(24)

where a and b are empirical coefficients with units and respectively and is the wind speed measured approximately 2 meters above the water surface . Several authors have proposed values for these coefficients including Dunne and Leopold (1978), who proposed the values and for the coefficients a and b respectively. These values are used as the defaults in the RHEComponent but can be overridden by user specified coefficients.

## Convective and Conductive Heat Fluxes

Estimating heat lost or gained through conduction/convection with air in the atmosphere is typically performed using the Bowen ratio , which relates latent heat to sensible heat (Equation 25) (Bowen, 1926; Webb and Zhang, 1997; Evans *et al.*, 1998; Westhoff *et al.*, 2007; Glose *et al.*, 2017).

(25)

The Bowen ratio is estimated as (Evans *et al.*, 1998; Westhoff *et al.*, 2007; Glose *et al.*, 2017):

(26)

where and are water and air temperature respectively and is the adiabatic atmospheric pressure. is computed as (Westhoff *et al.*, 2007; Glose *et al.*, 2017):

(27)

where is the elevation above sea level .

## Longwave Back radiation

Longwave back radiation from the stream surface is calculated from the attenuated form of the Stefan- Boltzmann law (McCutcheon, 1990) as follows:

(28)

where is the emissivity of the material, is water temperature in the channel , and is the Stefan-Boltzmann constant .

## Solvers

The RHEComponent solves Equation 4 using several ordinary differential equation (ODE) solvers including the classical fourth order Runge-Kutta method (i.e., RK4) or the adaptive step size controlled fifth order Runge-Kutta-Cash-Carp (RKQS, Cash and Karp, 1990) method. Alternatively, users can select variable multistep methods including the Adams-Moulton (i.e., ADAMS) formulas or the Backward Differentiation Formulas (i.e., BDF) that are provided through the CVODE (Hindmarsh *et al.*, 2017) external ODE solver library.

# Input File Format

The RHEComponent input file format is illustrated below. Values can be separated by space, tab, or comma delimiters. Delimiters can be any length.

[OPTIONS]

START\_DATETIME 5/01/2017 0:00:00

END\_DATETIME 5/10/2017 0:00:00

REPORT\_INTERVAL 900.0

MAX\_TIME\_STEP 60.0

MIN\_TIME\_STEP 0.00001

NUM\_INITIAL\_FIXED\_STEPS 10

USE\_ADAPTIVE\_TIME\_STEP YES

TIME\_STEP\_RELAXATION\_FACTOR 0.9

ADVECTION\_MODE UPWIND

COMPUTE\_DISPERSION NO

TEMP\_SOLVER ADAMS

TEMP\_SOLVER\_ABS\_TOL 1e-10

TEMP\_SOLVER\_REL\_TOL 1e-6

WATER\_DENSITY 1000.0

WATER\_SPECIFIC\_HEAT\_CAPACITY 4184

EVAPORATION YES

CONDENSATION YES

NUM\_SOLUTES 1

VERBOSE YES

FLUSH\_TO\_DISK\_FREQ 1000

PRINT\_FREQ 1000

[OUTPUTS]

;;OUTPUT\_TYPE FILEPATH

;;=======================

CSV ./green\_river\_test2.csv

NETCDF ./green\_river\_test2.nc

[SOLUTES]

;;SOLUTE\_NAME SOLVER\_TYPE SOLVER\_ABS\_TOL SOLVER\_REL\_TOL

;;===========================================================

Salinity ADAMS 1E-10 1E-8

[ELEMENTJUNCTIONS]

;;JUNCTION X Y Z

;;====================================

J1 0.0 0.0 1717.0

J2 1000.0 0.0 1717.0

J3 2000.0 0.0 1717.0

[ELEMENTS]

;;ELEMENT FROMJUNCTION TOJUNCTION LENGTH DEPTH XSECTION\_AREA WIDTH SLOPE FLOW DISPERSION\_COEFF TEMPERATURE SOLUTE1\_CONC

;;================================================================================================================================

E1 J1 J2 1000 0.1 5.4 54 0.0021 0.010 0.000 5.500 0.1

E2 J2 J3 1000 0.1 5.4 54 0.0021 0.010 0.000 5.500 0.1

[UNIFORM\_HYDRAULICS]

;;START\_ELEMENT END\_ELEMENT VARIABLE TYPE VALUE/FILEPATH

;;==================================================================

[NON\_UNIFORM\_HYDRAULICS]

;;VARIABLE FILEPATH

;;==================

FLOW ./flow.csv

DEPTH ./depth.csv

XSECTION\_AREA ./xsection\_area.csv

[BOUNDARY\_CONDITIONS]

;;JUNCTION VARIABLE TYPE VALUE/FILEPATH

;;============================================

J1 TEMPERATURE FILE ./hw\_temp.csv

[POINT\_SOURCES]

;;ELEMENT VARIABLE TYPE VALUE/FILEPATH

;;====================================================

E104 TEMPERATURE FILE ./yampa\_temp\_flux.csv

[NON\_POINT\_SOURCES]

;;START\_ELEMENT START\_ELEMENT\_LFACTOR END\_ELEMENT END\_ELEMENT\_LFACTOR VARIABLE TYPE VALUE/FILEPATH

;;=================================================================================================================

[UNIFORM\_RADIATIVE\_FLUXES]

;;START\_ELEMENT END\_ELEMENT TYPE VALUE/FILEPATH

;;======================================================

E1 E150 FILE ./rad\_flux.csv

[NON\_UNIFORM\_RADIATIVE\_FLUXES]

;;FILEPATH

;;==================

[UNIFORM\_METEOROLOGY]

;;START\_ELEMENT END\_ELEMENT VARIABLE TYPE VALUE/FILEPATH

;;==========================================================================

E1 E150 RELATIVE\_HUMIDITY FILE ./rel\_humid.csv

E1 E150 AIR\_TEMPERATURE FILE ./air\_temp.csv

E1 E150 WIND\_SPEED FILE ./wind\_speed.csv

[NON\_UNIFORM\_METEOROLOGY]

;;VARIABLE FILEPATH

;;====================

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